# Lab 5

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November 20, 2016

### 1 Introduction

The lab requires you to code the NEH heuristic. The NEH heuristic was developed by Nawaz, Enscore, and Ham and published in 1983 [1] and for many years has been commonly regarded as the best heuristic for solving the permutation flow shop problem. A more general problem is defined as follows. A set of n jobs,  $\{1, 2, ..., n\}$ , available at time zero has to be processed in a shop with m (ordered) machines  $\{M_1, M_2, ..., M_m\}$ . Each job is processed first on  $M_1$ , next on  $M_2$ , and so on, and lastly on  $M_m$ . No machine can process more than one job at a time, no preemption is allowed, all setup times are included into the job processing times, and there is unlimited storage between the machines. The problem, commonly referred to as  $F_m \parallel C_{\max}$ , is to determine a schedule that minimizes the completion time of the last job on  $M_m$ , also known as the **makespan**. The schedule with the same job ordering on every machine is called a permutation schedule, and the related problem,  $F_m | \text{prmu} | C_{\text{max}}$ , is to find a job sequence that minimizes the makespan. For m = 2 and 3, the search of the optimal schedule can be restricted to permutation schedules, but the optimal schedules may have different job orderings on different machines when m > 3. The  $F_3$  prmu  $|C_{\text{max}}|$  is strongly NP-hard. The  $F_2$  prmu  $|C_{\text{max}}|$  can be solved in  $O(n \log n)$  time by the well known algorithm of Johnson [2].

#### 2 The NEH heuristic

Let  $p_{ij}$  be the processing time of job j on machine  $M_i$  for i = 1, 2, ..., m and j = 1, 2, ..., n. The makespan,  $C_{\max}(\pi)$ , of a job sequence  $\pi = (\pi(1), \pi(2), ..., \pi(n))$  can be represented by the length of a critical (longest) path in an acyclic network. The network for computing  $C_{\max}(\pi)$ , where for simplicity  $\pi = (1, 2, ..., n)$ , is depicted in Fig. 1; the nodes are numbered by the corresponding job processing times. The makespan can be determined in O(mn) time by the critical path method (CPM).

The NEH heuristic can be described as follows:

**Step 1** : processing times 
$$T_k = \sum_{i=1}^m p_{ik}$$

**Step 2** : Take the first two jobs, find their order with the shorter makespan, and set L = 3.



Fig. 1. Network for computing the makespan of  $C_{\max}(\pi)$ , where  $\pi = (1, 2, ..., n)$ .

**Step 3** : Assume the current subsequence is  $(j_1, j_2, ..., j_{L-1})$ , and the  $L^{th}$  job determined in Step 1 is r. Among the L subsequences  $(r, j_1, j_2, ..., j_{L-1}), (j_1, r, j_2, ..., j_{L-1}), ..., (j_1, j_2, ..., j_{L-1}, r)$  find that one with the shortest makespan.

**Step 4**. Set L =: L + 1. If L = n + 1, then stop; otherwise return to Step 3.

If one assumes that each of the L makespans computed in Step 3 needs O(mL) operations, then NEH requires  $O(mn^3)$  time [3].

### **3** Experimentation

The student is required to code the NEH algorithm and solve the three different variants of scheduling problems of **flowshop** (FSS), **flowshop with blocking** (FSSB) and **flowshop** with no-wait (FSSNW) constraints for the the Taillard data sets. The makespan calculations for all three problems are provided in C++ codes, and the student is required to incorporate it in their own development framework.

### Submission

The student must submit the following separate files to canvas:

- 1. source codes for the problems
- 2. a LATEX typeset report on the results

The report must contain an introduction of NEH and the makespan values for all the problem instances for the three different problems has to be tabulated, alongside execution times.

The files must be submitted through Canvas by midnight December 5, 2016. No submission will be accepted after this date! The grading rubric is given in Table 1.

File	Aspects	Points
Code	Compiles and executes Explanation	$35 \\ 15$
Report	Results Analysis	$25 \\ 25$

Table 1: Grading rubric

## References

- Ham I. Nawaz M, Enscore Jr. E. A heuristic algorithm for the m-machine, n-job flowshop sequencing problem. Omega, the International Journal of Management Science, 11:91–95, 1983.
- [2] Johnson SM. Optimal two- and three-stage production schedules with setup times included. Naval Research Logistics Quarterly, 1(61-68), 1954.
- [3] Kalczynski P and Kamburowski J. On the neh heuristic for minimizing the makespan in permutation flow shops. *Omega, the International Journal of Management Science*, 35:53–60, 2007.