

# Project 3

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November 8, 2016

## 1 INTRODUCTION

For this lab we took our 15 optimization functions and ran them through 3 new methods of determining the global minimum. The functions being: Differential evolution (DE) which uses a population approach with strategies for computing new solutions, Genetic evolution (GE) which takes a genetic approach with genes, crossover and mutation, and Particle swarm (PS) which simulates swarm movement to find the global minimum.

## 2 METHODS

Our rewrite in the previous lab allowed us to just extend three more classes from the search function class we had implemented. These extended classes were then called with a python script and the output printed to the console where I was able to analyze the data. Each test was ran 15 times using the python script and the data stored to a file.

Upon referencing multiple online sources, we also decided to use a different method of initializing particle velocity vectors and velocity maximums. Settling with  $velocityMaximum = 4.0$ , and the initial velocities being created between  $velocityMaximum \cdot 3$ , and  $-velocityMaximum \cdot 3$ . These values seemed to produce accurate results.

## 3 ANALYSIS

This lab produced some interesting results regarding the performance of the new functions. Overall, the three new functions (PS, GE, DE) were more efficient and accurate than the previous 3 functions (Random Walk, Local Search, Iterative Local Search), but there were some

discrepancies with some functions. These discrepancies showed themselves as completely inaccurate results on some functions, while the method would then produce extremely accurate results on other functions. For example, GE produced a 23185.53 average for DeJong, the actual minimum being 0. Yet for the Michalewicz function GE produced an average that was much more competitive to the the other functions.

Another interesting point on the performance of these functions can be seen when comparing them to the values received from the previous search functions we used. Rosenbrocks saddle is a great example of the performance difference, where Iterative Local Search's best value was in the range of  $2.51E+10$ . DE on the other hand was able to produce a minimum value of 19 and PS a value of 37, massive increases in accuracy. Interestingly enough, for rosenbrocks saddle GE produced a value similar to Iterative local search, a minimum of  $3.21E+09$ .

More differences between the three functions can be again found with the Rastrigin function. ILS was able to produce a value of 83731.6, GE produced 65280, but PS and DE both had massively more accurate results of: PS -> -6902.05, and DE -> -8000 which we believe is the actual minimum of the Rastrigin function.

There are other examples of these new functions attaining greater accuracy than the previous functions did, but that information can easily be seen in the results table in figure 5.1. One last point we want to cover is the actual time performance of these algorithms. Previously Local Search and Iterative Local Search both took an excessive amount of time to compute on solutions with larger dimensions (20 +). Based on previous performance, it was estimated that the Griegwangk function running with 30 dimensions would run for 8 hours using Iterative Local Search. To contrast this, the complete computation time taken for the 15 functions, at 15 iterations, using all 3 search functions completed faster than one iteration of 20 dimensional Iterative Local Search with the Griewagnk function.

## 4 CONCLUSION

Coming away from this lab we saw that these new functions have the ability to not only improve the accuracy of our results, but also improve the running time of the search. With this improved running time we could run more trials and get even more accurate results than the ones that we are getting currently.

There were some difficulties and issues when running the tests for this lab. The first being our inability to completely verify our results. We mentioned some discrepancies earlier where GE produced values that were wildly inaccurate for some functions. It is unknown to us whether this is simply a product of the strengths and weaknesses of this specific search method, or if there is something wrong with out implementation. Another issue is that of the Shekels Foxholes function. For Particle Swarm and Genetic Evolution there was no deviation from the single value that they returned. Either the algorithm is able to deterministic produce the apparent global minimum, or there is something wrong with the function.

## 5 RESULTS

Figure 5.1: Computation comparison of DE, GA and PSO

| Problem  | DE       |          |           |          |      | GA            |               |                |               |       | PSO      |          |         |         |      |
|----------|----------|----------|-----------|----------|------|---------------|---------------|----------------|---------------|-------|----------|----------|---------|---------|------|
|          | Avg      | Median   | Range     | SD       | T(s) | Avg           | Median        | Range          | SD            | T(s)  | Avg      | Median   | Range   | SD      | T(s) |
| $f_1$    | -6112.33 | -6084.59 | 114.26    | 47.83    | 1.14 | -3276.12      | -3292.95      | 943.02         | 245.68        | 2.69  | -2871.98 | -2904.39 | 1194.77 | 322.06  | 0.12 |
| $f_2$    | 129.53   | 25.00    | 900.00    | 251.52   | 0.53 | 23185.53      | 22853.00      | 10310.00       | 3148.43       | 0.72  | 0.17     | 0.15     | 0.25    | 0.08    | 0.09 |
| $f_3$    | 26105.67 | 10019.00 | 168100.00 | 43662.88 | 0.78 | 5291234666.67 | 5017400000.00 | 57390200000.00 | 1539343402.74 | 0.68  | 421.98   | 200.19   | 1657.68 | 497.31  | 0.10 |
| $f_4$    | -7600.00 | -7960.00 | 2560.00   | 728.99   | 1.00 | 79752.00      | 81520.00      | 23240.00       | 8507.40       | 2.12  | -5206.62 | -5324.98 | 3479.78 | 1178.83 | 0.13 |
| $f_5$    | 0.00     | 0.00     | 0.00      | 0.00     | 1.08 | 145.86        | 150.55        | 51.89          | 17.68         | 2.31  | 9.17     | 8.93     | 5.88    | 1.95    | 0.13 |
| $f_6$    | 12.38    | 12.71    | 2.19      | 0.60     | 1.46 | 12.04         | 11.97         | 0.67           | 0.22          | 2.52  | 12.15    | 12.18    | 1.25    | 0.33    | 0.14 |
| $f_7$    | 19.06    | 19.01    | 0.62      | 0.16     | 1.67 | 36.69         | 36.60         | 5.76           | 1.54          | 4.20  | 20.55    | 20.45    | 2.63    | 0.68    | 0.18 |
| $f_8$    | 58.74    | 58.73    | 4.74      | 1.54     | 1.60 | 212.86        | 213.95        | 41.20          | 11.06         | 3.41  | -9.92    | -11.64   | 35.51   | 9.72    | 0.10 |
| $f_9$    | -83.30   | -80.69   | 21.87     | 6.99     | 2.09 | 276.38        | 276.83        | 14.65          | 4.35          | 4.10  | 251.53   | 288.37   | 173.05  | 64.83   | 0.14 |
| $f_{10}$ | -4959.12 | -4579.12 | 2896.23   | 966.10   | 3.02 | -4778.37      | -4822.17      | 978.82         | 327.79        | 4.72  | -4107.05 | -3830.50 | 2663.98 | 711.61  | 0.13 |
| $f_{11}$ | -8478.48 | -8821.20 | 5161.40   | 1330.20  | 3.56 | -3188.30      | -3181.83      | 1334.30        | 339.30        | 8.34  | -2899.33 | -2888.72 | 901.67  | 227.81  | 0.21 |
| $f_{12}$ | 0.00     | 0.00     | 0.00      | 0.00     | 1.48 | 8.00          | 8.01          | 0.69           | 0.17          | 2.70  | 7.02     | 7.08     | 1.30    | 0.37    | 0.15 |
| $f_{13}$ | -4.28    | -4.22    | 2.71      | 0.83     | 3.06 | -4.27         | -4.22         | 2.30           | 0.57          | 5.54  | -10.39   | -9.86    | 4.92    | 1.50    | 0.14 |
| $f_{14}$ | -18.99   | -19.00   | 0.04      | 0.01     | 1.47 | -10.88        | -10.53        | 3.70           | 1.00          | 3.65  | -16.07   | -16.15   | 5.22    | 1.59    | 0.14 |
| $f_{15}$ | -21.91   | -23.03   | 8.39      | 2.95     | 6.05 | -14.64        | -14.64        | 0.00           | 0.00          | 12.55 | -18.70   | -18.70   | 0.00    | 0.00    | 0.27 |

<sup>1</sup> ThinkPad, 3.4GHz Intel Core i7, 8GB RAM

# 6 PREVIOUS RESULTS

| Function  | 1                 | 2            | 3                | 4               | 5           | 6            | 7            | 8             | 9            | 10              | 11             | 12           | 13           | 14           | 15           |
|-----------|-------------------|--------------|------------------|-----------------|-------------|--------------|--------------|---------------|--------------|-----------------|----------------|--------------|--------------|--------------|--------------|
|           | -4549.4           | 0.0605       | 2.51E+10         | 83731.6         | 0.651206    | 14.7944      | 21.9681      | 279.695       | 321.001      | -1566.84        | -5338.46       | 9.21621      | -0.142619    | -17.715      | -11.8869     |
|           | -6050.16          | 0.0605       | 4.55E+10         | 80745.8         | 0.268086    | 16.2884      | 19.3337      | 343.725       | 313.969      | -1993.75        | -4148.75       | 9.46446      | -0.548652    | -18.1772     | -11.5925     |
|           | -5398.23          | 0.0591995    | 5.06E+10         | 82401.3         | 0.032413    | 15.3526      | 23.9768      | 296.889       | 321.956      | -4633.19        | -4848.21       | 9.01646      | 0.218791     | -18.4282     | -11.5925     |
|           | -5675.22          | 0.0605       | 4.42E+10         | 82591.3         | 0.00544314  | 14.7622      | 22.6234      | 331.276       | 323.928      | -4014.04        | -5436.05       | 9.26537      | 0.626347     | -18.2969     | -12.1455     |
|           | -3976.37          | 0.0588747    | 3.25E+10         | 82036.8         | 0.0128344   | 13.9223      | 25.6647      | 325.015       | 329.982      | -1246.54        | -4823.87       | 9.12409      | -1.33987     | -17.3054     | -11.5925     |
|           | -5082.38          | 0.0605       | 2.61E+10         | 87382.5         | 0.0201434   | 14.3422      | 21.3887      | 396.302       | 326.905      | -392.775        | -6280.62       | 9.13477      | -2.36781     | -18.1703     | -11.5925     |
|           | -5891.86          | 0.0605       | 3.65E+10         | 89279.1         | 0.684922    | 15.2594      | 23.0754      | 329.302       | 326.381      | -337.14         | -3871.16       | 9.02972      | -0.0212582   | -18.4513     | -11.5925     |
|           | -5003.93          | 0.0605       | 3.00E+10         | 85879           | 0.0151074   | 14.8268      | 19.9157      | 325.527       | 314.19       | -1212.15        | -4189.52       | 9.17302      | 0.0681864    | -18.2963     | -11.5925     |
|           | -5418.57          | 0.0605       | 4.30E+10         | 82890.7         | 0.00544314  | 15.9129      | 20.6394      | 332.571       | 325.788      | -1548.21        | -5548.41       | 9.39065      | -0.906265    | -18.5396     | -11.5925     |
|           | -5516.7           | 0.0599162    | 3.51E+10         | 82665.9         | 0.03008     | 16.1692      | 19.9074      | 388.651       | 327.169      | 923.714         | -563.104       | 9.20288      | -1.59629     | -18.1054     | -11.5925     |
|           | -3937.92          | 0.0547374    | 3.35E+10         | 86354.3         | 0.0153078   | 15.3926      | 19.532       | 312.15        | 322.111      | -1598.94        | -4648.01       | 8.87789      | -1.49817     | -17.8506     | -11.5925     |
|           | -4588.88          | 0.0605       | 3.51E+10         | 81101.4         | 0.00542657  | 15.7841      | 21.7808      | 357.263       | 330.684      | -1319.67        | -4261.9        | 9.33048      | -1.93719     | -18.2627     | -12.1791     |
|           | -5082.97          | 0.0559769    | 2.42E+10         | 76218.4         | 0.00544314  | 15.1429      | 22.4065      | 346.209       | 324.783      | 1500.62         | -6032.9        | 9.18852      | -1.88437     | -18.2965     | -11.5925     |
|           | -6070.01          | 0.0594556    | 3.21E+10         | 83486.2         | 0.0201913   | 14.966       | 19.342       | 393.145       | 324.682      | -404.273        | -5094.61       | 8.57302      | 0.126058     | -16.446      | -11.5925     |
|           | -5043.31          | 0.0605       | 1.85E+10         | 82935.5         | 0.220955    | 14.0295      | 20.2134      | 316.233       | 325.996      | 2079.2          | -4572.43       | 9.70061      | -1.33897     | -18.2227     | -11.5925     |
|           | -5161.34          | 0.0605       | 3.87E+10         | 85337.7         | 0.00541984  | 14.9478      | 19.5923      | 274.232       | 327.758      | -1727.17        | -4684.33       | 9.01989      | 0.290542     | -18.4796     | -12.1791     |
|           | -4589.3           | 0.060484     | 3.08E+10         | 83765.5         | 0.01776     | 14.714       | 20.0449      | 314.196       | 325.896      | -1632.08        | -5179.06       | 8.52944      | -0.524841    | -18.3305     | -18.4163     |
|           | -4332.6           | 0.0600202    | 6.77E+10         | 81498.7         | 0.0153003   | 16.5447      | 20.0096      | 314.165       | 307.423      | 1567.7          | -4461.73       | 9.43874      | -1.20987     | -18.3268     | -11.5925     |
|           | -6267.41          | 0.0605       | 4.65E+10         | 81603.3         | 1.88596     | 16.1234      | 20.2482      | 316.52        | 317.494      | -5550.9         | -3683.39       | 9.4962       | 0.510028     | -18.1621     | -11.5925     |
|           | -4588.98          | 0.0605       | 4.06E+10         | 85076.2         | 0.0225431   | 14.0527      | 20.8396      | 325.67        | 324.854      | -2326.43        | -5346.36       | 9.33666      | -1.32004     | -18.555      | -11.5925     |
|           | -5477.69          | 0.0604347    | 5.13E+10         | 80799.5         | 0.00535823  | 14.7108      | 20.8015      | 261.241       | 319.17       | 485.619         | -4782.73       | 9.17792      | -1.22079     | -17.7232     | -18.1189     |
|           | -6109.42          | 0.0472961    | 2.59E+10         | 83373.9         | 0.177322    | 15.6127      | 22.9987      | 286.388       | 321.812      | -2998.41        | -4456.01       | 8.6997       | -2.29986     | -18.4625     | -11.5925     |
|           | -4885.09          | 0.0570039    | 4.39E+10         | 80163.4         | 0.255374    | 15.6927      | 23.8355      | 326.354       | 325.002      | -1578.78        | -4075.51       | 9.10547      | -0.131814    | -17.7901     | -11.5925     |
|           | -4035.51          | 0.0600693    | 4.32E+10         | 81415.3         | 0.0152417   | 14.9575      | 19.5572      | 296.019       | 321.434      | 824.034         | -4011.91       | 9.31902      | -1.51983     | -17.8755     | -11.5924     |
|           | -3917.22          | 0.0588818    | 4.16E+10         | 87878.4         | 0.0128369   | 14.4221      | 19.8007      | 375.301       | 300.768      | 1043.95         | -5037.25       | 9.57629      | -1.73324     | -18.2571     | -11.5925     |
|           | -5754.04          | 0.0575467    | 2.76E+10         | 83622.3         | 0.272711    | 17.8043      | 20.3977      | 370.717       | 317.424      | -1133.68        | -4973.3        | 9.0252       | -1.10353     | -18.3087     | -11.5925     |
|           | -4786.87          | 0.0605       | 5.54E+10         | 81190           | 0.0324796   | 14.4401      | 19.8204      | 358.965       | 296.103      | 692.475         | -421.91        | 8.76355      | -0.860885    | -18.484      | -18.4163     |
|           | -4510.47          | 0.0597792    | 3.24E+10         | 83530.5         | 4.0042      | 16.0272      | 20.4454      | 225.399       | 326.39       | -478.625        | -5370.33       | 8.33801      | -1.13941     | -18.4418     | -11.5729     |
|           | -5398.26          | 0.0605       | 2.12E+10         | 8510.6          | 0.48707     | 14.8375      | 19.3774      | 332.968       | 327.903      | -985.672        | -316.499       | 8.94067      | 0.336942     | -17.9256     | -11.5925     |
|           | -4253.69          | 0.0600716    | 3.11E+10         | 84551           | 0.0128391   | 15.5474      | 21.1043      | 361.128       | 322.473      | -2662.15        | -459.38        | 7.86002      | -0.784451    | -18.5094     | -11.5924     |
| Avg.      | -5045.12666666667 | 0.05921826   | 3.70E+10         | 83304.4         | 3.06E-01    | 1.52E+01     | 2.10E+01     | 3.27E+02      | 3.21E+02     | -1.07E+03       | -4.51E+03      | 9.11E+00     | -8.39E-01    | -1.81E+01    | -1.23E+01    |
| Med.      | -5062.845         | 0.06045935   | 35130150000      | 83154.7         | 0.02016735  | 15.05445     | 20.33055     | 325.5985      | 323.955      | -1229.345       | -4733.53       | 9.17547      | -1.0048975   | -18.2599     | -11.5925     |
| Std. Dev. | 698.894252189     | 0.0026900942 | 10975111347.1528 | 2619.8872400694 | 0.791604351 | 0.8539153929 | 1.6317417953 | 39.4394076324 | 8.0216605796 | 1808.4389363295 | 1268.809305736 | 0.3628603461 | 0.8562948838 | 0.4364346365 | 2.0376857083 |

## Iterative Local Search Running Times in Seconds

| Dimensions  | 10             | 20              | 30             |
|-------------|----------------|-----------------|----------------|
| Function 1  | 5.355587244    | 21.5247523785   | 47.5882720947  |
| Function 2  | 0.4999251366   | 1.151144743     | 2.4649145603   |
| Function 3  | 0.0042607784   | 0.0112228394    | 0.0144929886   |
| Function 4  | 0.0058951378   | 0.0114533901    | 0.0161828995   |
| Function 5  | 150.1059572697 | 2928.3961615563 | N/A            |
| Function 6  | 0.0101454258   | 0.0255510807    | 0.0222308636   |
| Function 7  | 32.4964332581  | 41.5021996498   | 168.8056237698 |
| Function 8  | 0.3526818752   | 1.6770370007    | 3.8826031685   |
| Function 9  | 0.0110986233   | 0.0125215054    | 0.0229070187   |
| Function 10 | 0.0899729729   | 0.2644715309    | 0.7342042923   |
| Function 11 | 30.093629837   | 165.0208876133  | 384.6772966385 |
| Function 12 | 297.458874464  | 25.3617525101   | 21.2174470425  |
| Function 13 | 0.0197796822   | 0.0436241627    | 0.0463643074   |
| Function 14 | 1.5726833344   | 7.7616007328    | 9.7854065895   |
| Function 15 | 6.6486163139   | 23.9164574146   | 32.7224471569  |